AD-756 829

RECEIVER NOISE DUE TO R/V PLASMA SHEATH TURBULENCE

Ronald L. Fante

Air Force Cambridge Research Laboratories
L. G. Hanscom Field, Massachusetts

18 December 1972

Best
Available
Copy

AFCRL-72-0735
18 DECEMBER 1972
PHYSICAL SCIENCES RESEARCH PAPERS, NO. 520

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES

L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

Receiver Noise Due to R/V Plasma Sheath Turbulence

RONALD L. FANTE

AIR FORCE SYSTEMS COMMAND
United States Air Force

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Scatter by turbulence						
Turbulent noise						

Unclassified

Security Classification

Contents

Illustrations

Tables

Preceding page blank

/

Receiver Noise Due to R/V Plasma Sheath Turbulence

1. INTRODUCTION

In some reentry applications, it is important to know the noise power which will be introduced into an antenna by the turbulent plasma surrounding the vehicle. In this report we will calculate the noise power backscattered into an antenna by an underdense, turbulent plasma. The starting point for this calculation is the transport equation for the radiative intensity derived by Watson (1969, 1970). This is

$$\hat{p} \cdot \nabla [I(\underline{x}, p)] + \left(\frac{1}{\ell_a} + \frac{1}{\ell_t}\right)[I] = \int d\Omega' [M(\hat{p}, \hat{p}')] [I(\underline{x}, \hat{p}')] \ . \tag{1}$$

The term [I] is a column matrix with four components, as defined in Eq. (29) of Watson (1970), and the components $I_j(\underline{x}, \hat{p})d\Omega$ represent the radiant energy per unit area travelling in the direction $\hat{p}$ within the element of solid angle $d\Omega$, with the appropriate polarization properties. The term ℓ_a is the mean free path for collisional absorption, ℓ_t is the mean free path for turbulent scatter, and [M] is a 4×4 matrix with components given in Table II of the paper by Watson (1970).

(Received for Publication 18 December 1972)

2. THEORETICAL CONSIDERATIONS

To study Eq. (1) further, it is convenient to separate [I] into an unscattered portion $[I^o]$, and a scattered component $[I^s]$. We then have, from Eq (1)

$$\hat{p} \cdot \nabla [I^o] + \left(\frac{1}{\ell_a} + \frac{1}{\ell_t}\right)[I^o] = 0 \tag{2}$$

$$\hat{p} \cdot \nabla [I^s] + \left(\frac{1}{\ell_a} + \frac{1}{\ell_t}\right)[I^s] = \int d\Omega' [M][I^o] + \int d\Omega' [M][I^s] \ . \tag{3}$$

Now if we consider backscatter from a layer of thickness $L \ll \ell_t$, the component $I^s \ll I^o$ and the second term on the right-hand side of Eq. (3) can be neglected in comparison with the first. For the reentry sheath around many vehicles this approximation is usually valid. A tabulation of ℓ_t as a function of the turbulent correlation length, a, and the relative fluctuation, $\delta\omega_p/\omega$, in the plasma frequency, ω_p, is given in Table 1, for an X-band transmitter frequency. Since the plasma sheath thickness is generally of order 10 cm, it is clear from Table 1 that ℓ_t is generally much greater than the sheath thickness, so that $[I^s]$ satisfies

$$\hat{p} \cdot \nabla [I^s] + \frac{1}{\ell}[I^s] \simeq \int d\Omega' [M][I^o] \tag{4}$$

where $1/\ell = 1/\ell_a + 1/\ell_t$.

Table 1. Turbulent Mean Free Path for X-band (9.6 GHz)

a = 0.25 cm		a = 0.79 cm		a = 5 cm	
ℓ_t(cm)	$\delta\omega_p/\omega$	ℓ_t(cm)	$\delta\omega_p/\omega$	ℓ_t(cm)	$\delta\omega_p/\omega$
6.08×10^8	0.01	9.4×10^7	0.01	1.12×10^7	0.01
6.08×10^4	0.1	9.4×10^3	0.1	1.12×10^3	0.1
97	0.5	15	0.5	70	0.2

To apply Eq. (4) to the reentry problem, we assume that the surface of the radiating antenna lies in the z = 0 plane, and that the antenna is covered by a homogeneous turbulent plasma layer extending from z = 0 to z = L. We also assume that the unscattered intensity is a very narrow angle beam which can be approximated for $z < \ell_t$ by

$$[I^{o}(\underline{x},\hat{p})] = [I(z=0,\underline{r}_{\perp})]\, e^{-z/\ell}\, \delta_{\hat{z},\hat{p}} \tag{5}$$

where $\underline{r}_{\perp}$ is the coordinate transverse to the z-axis and $\hat{z}$ is a unit vector along the z-axis. Upon using Eq. (5) in Eq. (4), we obtain

$$(\hat{p}\cdot\nabla)[I^{s}] + \frac{1}{\ell}[I^{s}] = [M(\hat{p},\hat{z})]\,[I(0,\underline{r}_{\perp})]\, e^{-z/\ell}\,. \tag{6}$$

Equation (6) may be solved by Fourier transforms. If we write

$$[I^{s}] = \iint_{-\infty}^{\infty} d^{2}k\, e^{-i\underline{k}\cdot\underline{r}_{\perp}}\, [F(z,\underline{k},\hat{p})] \tag{7}$$

we obtain for [F] :

$$\frac{\partial[F]}{\partial z} + \alpha[F] = \frac{e^{-z/\ell}}{\mu}[M(\hat{p},\hat{z})]\,[\hat{I}(0,\underline{k})] \tag{8}$$

where $\mu = \cos\theta$, θ is the angle between $\hat{p}$ and the z axis as shown in Figure 1, and $\hat{I}(0,\underline{k})$ is the Fourier transform of $I(0,\underline{r}_{\perp})$. Also

$$\alpha = \frac{1}{\ell\mu} - i\frac{\underline{k}\cdot\hat{p}_{\perp}}{\mu}\,.$$

Equation (8) is readily solved for F. The result is, for $-1 \le \mu \le 0$

$$[F(z,\underline{k},\hat{p})] = -[M(\hat{p},\hat{z})]\,[\hat{I}(0,\underline{k})]\,\frac{e^{-\alpha z}}{\mu}\int_{z}^{L} dz'\, e^{(\alpha-1/\ell)z'}\,. \tag{9}$$

In deriving Eq. (9) we have used the fact that F(z = L) is zero for $-1 \le \mu \le 0$, since there can be no scattered radiation at z = L in the backward hemisphere.

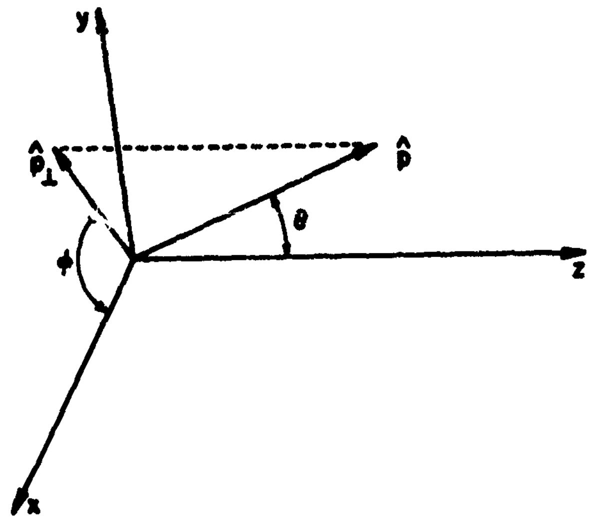

Figure 1. Orientation of the $\hat{p}$ Vector

Now let us suppose that tnere is a receiving aperture in the z = 0 plane with a response $A(\underline{r}_\perp)$. Then the power received by this aperture is

$$[P_{rec}] = (2\pi)^2 \int_{-1}^{0} |\mu| \, d\mu \int_0^{2\pi} d\phi \int d^2k \, \hat{A}(\underline{k}) [F(0, \underline{k}, \mu, \phi)] \tag{10}$$

where $\hat{A}$ is the Fourier transform of $A(\underline{r}_\perp)$, and ϕ is the azimuthal angle as shown in Figure 1. To evaluate Eq. (10) we assume for mathematical simplicity that

$$[I(0, \underline{r}_\perp)] = \frac{1}{S} [P_o] \exp\left(- \frac{\pi r_\perp^2}{S} \right) \tag{11}$$

$$A(r_\perp) = \exp\left(- \pi r_\perp^2 / \Sigma_o \right) \tag{12}$$

so that

$$[\hat{I}(0, \underline{k})] = (2\pi)^{-2} [P_o] \exp\left(- Sk^2/4\pi\right) \tag{13}$$

$$\hat{A}(\underline{k}) = (2\pi)^{-2} \Sigma_o \exp\left(-\Sigma_o k^2/4\pi\right) . \tag{14}$$

The aperture response $A(\underline{r}_\perp)$ assumed in Eq. (12) is, of course, not realistic but is convenient for mathematical evaluation. Upon using Eqs. (9), (13) and (14) in Eq. (10), we obtain after writing $d^2k = k dk d\psi$ and performing the ψ integration

$$[P_{rec}] = -\int_{-1}^{0} |\mu| d\mu \int_0^{2\pi} d\phi \, \frac{\Sigma_0}{2\pi\mu} [M(\hat{p}, \hat{z})] [P_0] \int_0^L dz' \, e^{\frac{z'}{\ell}\left(\frac{1}{\mu} - 1\right)}$$

$$\times \int_0^\infty k dk \, J_0\left(\frac{z' |\hat{p}_\perp| k}{\mu}\right) d^{-qk^2} \tag{15}$$

where $q = (S+\Sigma_0)/4\pi$. The k integration in Eq. (15) is readily performed (Erdelyi, 1954; p. 185) to give

$$[P_{rec}] = -\frac{\Sigma_0}{4\pi q} \int_{-1}^{0} \frac{|\mu|}{\mu} d\mu \int_0^{2\pi} d\phi \, [M(\hat{p}, \hat{z})] [P_0] \int_0^L dz' \, e^{\frac{z'}{\ell}\left(\frac{1}{\mu} - 1\right)}$$

$$\times \, e^{-\frac{z'^2 |\hat{p}_\perp|^2}{4\mu^2 q}} \, . \tag{16}$$

The integration on z' is readily performed (Abramowicz and Stegun, 1964, p. 303), and Eq. (16) becomes

$$[P_{rec}] = R \int_{-1}^{0} |\mu| d\mu \int_0^{2\pi} d\phi \, T(\mu) Q(\mu) [M(\hat{p}, \hat{z})] [P_0] \tag{17}$$

where

$$R = \Sigma_0 / 4(\pi q)^{1/2}$$

$$T(\mu) = (1-\mu^2)^{-1/2} \exp\left[\frac{(1+|\mu|)q}{\ell^2(1-|\mu|)}\right]$$

$$Q(\mu) = \mathrm{erf}\left[\left(\frac{1-\mu^2}{q}\right)^{1/2} \frac{L}{2|\mu|} + \frac{1}{\ell}\left(\frac{1+|\mu|}{1-|\mu|} q\right)^{1/2}\right]$$

$$- \, \mathrm{erf}\left[\frac{1}{\ell}\left(\frac{1+|\mu|}{1-|\mu|} q\right)^{1/2}\right].$$

Now, for an exponential correlation function it can be shown that

$$[M] = \sigma_g(\mu)[M_o] \tag{18}$$

where

$$\sigma_g(\mu) = \frac{A}{\left[\left(1+\frac{B}{2}\right) - \frac{B}{2}\mu\right]^2} \tag{19}$$

$$A = \frac{8\pi r_o^2 \langle \delta n_e^2 \rangle a^3}{(1+\nu_c^2/\omega^2)}$$

$$B = 4\left(\frac{2\pi a}{\lambda}\right)^2$$

and r_o is the classical electron radius, ν_c is the electron-neutral collision frequency, λ is the signal wavelength, ω is the radian signal frequency, and $\langle \delta n_e^2 \rangle$ is the mean-square electron density fluctuation. We note that for the exponential correlation function we obtain for ℓ_t, from Eq. (13a) of Watson (1970):

$$\ell_t = \frac{2\pi A}{1+B} - \frac{8\pi A}{B^3}\left\{- B + 2\left(1+\frac{B}{2}\right) \ell n\ (1+B) - \frac{B\left(1+\frac{B}{2}\right)^2}{1+B}\right\}. \tag{20}$$

Finally, for the matrix $[M_o]$ we have

$$[M_o] = \begin{bmatrix} \mu^2 \cos^2\phi & \mu^2 \sin^2\phi & \mu^2 \sin\phi \cos\phi & 0 \\ \sin^2\phi & \cos^2\phi & \frac{1}{2}\sin 2\phi & 0 \\ 2\mu \sin\phi \cos\phi & \mu \sin 2\phi & \mu \cos 2\phi & 0 \\ 0 & 0 & 0 & \mu \end{bmatrix} \tag{21}$$

If Eqs. (18), (19) and (21) are substituted into Eq. (17) and the ϕ integration performed, we obtain

$$[P_{rec}] = \pi R \int_{-1}^{0} |\mu| \, d\mu \, Q(\mu) T(\mu) \sigma_g(\mu) \begin{bmatrix} \mu^2 & \mu^2 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu \end{bmatrix} \begin{bmatrix} P_o^1 \\ P_o^2 \\ P_o^3 \\ P_o^4 \end{bmatrix} \tag{22}$$

The reader will now recall from the Watson (1970) paper that in the Stokes parameter representation employed, a linearly polarized incident wave has components $P_o^1 = P_{inc}$, but $P_o^2 = P_o^3 = P_o^4 = 0$. Therefore, if we assume the incident field is linearly polarized along the x-axis we have from Eq. (22)

$$\begin{bmatrix} P_{rec}^1 \\ P_{rec}^2 \\ P_{rec}^3 \\ P_{rec}^4 \end{bmatrix} = \pi R P_{inc} \int_{-1}^{0} |\mu| \, d\mu \, Q(\mu) \, T(\mu) \sigma_g(\mu) \begin{bmatrix} \mu^2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

If the receiving antenna is not polarization sensitive, the total received power will be the sum of the powers in each of the two polarizations; that is, $P_{rec} = P_{rec}^1 + P_{rec}^2$, which gives

$$\frac{P_{rec}}{P_{inc}} = \pi R \int_{-1}^{0} |\mu| \, d\mu \, Q(\mu) \, T(\mu) \, \sigma_g(\mu)(1+\mu^2) \, . \tag{24}$$

Before evaluating Eq. (24), it is convenient to substitute Eq. (19) for σ_g. After substituting for A in terms of ℓ_t from Eq. (20) and replacing μ by $-\mu$, we obtain:

$$\frac{P_{rec}}{P_{inc}} = \Lambda \int_{o}^{1} \frac{\mu d\mu \, Q(\mu) \, T(\mu)(1+\mu^2)}{\left[\left(1+\frac{B}{2}\right) + \frac{B}{2}\mu\right]^2} \tag{25}$$

where

$$\Lambda = \left(\frac{\pi^{1/2}\beta_o}{4}\right)\left(\frac{2\pi}{\sqrt{1+S/\Sigma_o}}\right)\left(\frac{a}{\ell_t}\right)\left(\frac{\Sigma_o}{\pi a^2}\right)^{1/2}$$

$$\beta_o = \frac{1}{2\pi}\left[\frac{1}{1+B} - \frac{4}{B^3}\left\{-B+2\left(1+\frac{B}{2}\right)\ln(1+B) - \frac{B\left(1+\frac{B}{2}\right)^2}{1+B}\right\}\right]^{-1} .$$

3. NUMERICAL RESULTS

The integral in Eq. (25) must be evaluated numerically. Let us suppose that $\delta\omega_p/\omega = 0.1$, $\Sigma_o = S = \pi\lambda^2/4$ where λ = signal wavelength, $\nu_c/\omega = 0.0$, and that the signal frequency is 9.6 GHz. Values of P_{rec}/P_{inc} are presented in Table 2 for L = 100 cm, and in Table 3 for L = 5 cm. We note from Table 3 that for correlation lengths of the order 1 cm, which is often estimated as the correlation length in reentry plasmas, the received turbulent noise is about 50 dB below the transmitted signal. Therefore, the turbulent noise will generally not present any obstacle to the operation of a one-way communication system. For two-way systems, however, such as a terminal guidance radar*, it is clear from our results that the noise generated by the plasma turbulence can be an important systems consideration. We have not attempted to do a full systems study at this time, but such a study is readily performed by using Eq. (25) with the appropriate systems parameters.

Table 2. Values of Received X-band Power When $\delta\omega_p/\omega = 0.1$, L = 100 cm $\nu_c/\omega = 0$

a(cm)	P_{rec}/P_{inc}
0.25	3.24×10^{-5}
0.79	1.15×10^{-5}
5	3.50×10^{-7}

* For such applications, the receiving and transmitting antennas will generally not be colocated, as has been assumed here. If the receiving and transmitting antennas are separated by a distance $\underline{r}_o$, we must replace Eq. (12) by

$$A(r_\perp) = \exp\left[-\pi(\underline{r}_\perp - \underline{r}_o)^2/\Sigma_o\right]$$

Table 3. Values of Received X-band Power When $\delta\omega_p/\omega = 0.1$, L = 5 cm, and $\nu_c/\omega = 0$

a (cm)	P_{rec}/P_{inc}
0.25	2.53×10^{-5}
0.79	0.927×10^{-5}
5	2.82×10^{-7}

Acknowledgment

I am grateful to Mr. Richard Taylor for his assistance in the numerical evaluation of Eq. (25).

References

Abramowicz, M. and Stegun, I. (1964) Handbook of Mathematical Functions, U.S. Government Printing Office, Washington, D.C.

Erdelyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F. (1954) Tables of Integral Transforms, Vol. 1, McGraw-Hill, New York.

Watson, K. (1969) Multiple scattering of electromagnetic waves in turbulent plasma, J. Math. Phys. **10**, 688-702.

Watson, K. (1970) Electromagnetic wave scattering within a plasma in the transport approximation, Phys. Fluids **13**, 2514-2523.